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MAGNETOPHORETIC PROPERTIES OF A VOLUME-ORDERED SYSTEM OF RECTANGULAR FERROCYLINDERS

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The magnetophoretic properties of a system of identical unidirectional rectangular ferromagnetic cylinders of infinite length have been considered. The influence of the geometric parameters of the system on the distribution of magnetophoretic potential and the characteristic time of isolation of dia- and paramagnetic particles has been studied.

The method of high-gradient magnetic separation has attracted attention in many spheres of activity, including water purification and gas scrubbing, clay cleaning, chemical technologies, medicine, and biology [1–7]. In actual practice, high-gradient magnetic filters are created by application of a strong homogeneous magnetic field to a volume containing a disordered charge of small ferromagnetic bodies. The possibility of improving the efficiency of such filters is very limited, particularly for solution of problems of separation of low-magnetic (dia- and paramagnetic) microparticles, including particles of cellular suspensions. Fundamentally new possibilities are offered, in our opinion, by the concept of high-gradient magnetic separation on coherent (ordered) magnetic structures [8–10]. Ordered structures allow detailed description of the distribution of a magnetic field in them, thus enabling one to use mathematical optimization methods. Furthermore, the order of a magnetic structure creates prerequisites for a sharp improvement in the properties of a filter by accumulation of the effect of magnetophoretic displacement of the particles isolated in the process of motion of the suspension. In the present work, we have considered the class of filter systems formed by volume-ordered sets of ferromagnetic cylinders of rectangular cross section. The magnetophoretic-potential method [8–10], which enables one to clearly visualize ideas required for practical designing of filters for various purposes, has been used as the basis for the investigation.

Geometry of the System and Initial Relations. The structure of a filter is shown in Fig. 1. Cylinders of infinite length are guided along the X axis and form a set of layers which lie in the planes perpendicular to the Z axis. The width of a cylinder is $2a$ and the height is $2B$; the structural step along Y is equal to S_y and that along Z is equal to S_z. We consider two variants of layering: in a rectangular order (I) and in an oblique order (II). For a complete description of a magnetic field in the periodic structure it is sufficient to consider regions bounded by the dashed lines in Fig. 1. A homogeneous external magnetic field $\mathbf{H}_0$ is guided either along the layering or across it perpendicularly to the cylinders' axes. We assume that the field strength H_0 is high and the cylinders are magnetized to saturation. The total strength of the magnetic field in the structure is equal to the sum of the external field and the eigenfield of the cylinders: $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}'$. The system to be separated represents a suspension of low-magnetic (dia- and paramagnetic) particles in a liquid or gaseous medium; the particle size is small as compared to the dimension of a cylinder. In this case the magnetic force acting on a particle is determined by the expression

$$\mathbf{F}_{\text{m}} = \frac{1}{2}\Delta\chi v \nabla \mathbf{H}^2 = -\nabla\Phi\,, \quad \Phi = -\frac{1}{2}\Delta\chi v\,(2\pi I_{\text{s}})^2\,[\mathbf{h}^2 + P\,(\mathbf{eh})]\,, \quad P = \frac{H_0}{\pi I_{\text{s}}}\,. \tag{1}$$

Here $\Delta\chi = \chi - \chi_0$, $\mathbf{h} = \mathbf{H}'/(2\pi I_{\text{s}})$ is the dimensionless eigenfield of the structure, and Φ is the magnetophoretic potential. Using the quantity $\Phi^* = 2\Delta\chi v(\pi I_{\text{s}})^2$ as the magnetophoretic-potential scale and $|\Phi^*|/a$ as the magnetophoretic-force scale, we rewrite relations (1) in dimensionless form:

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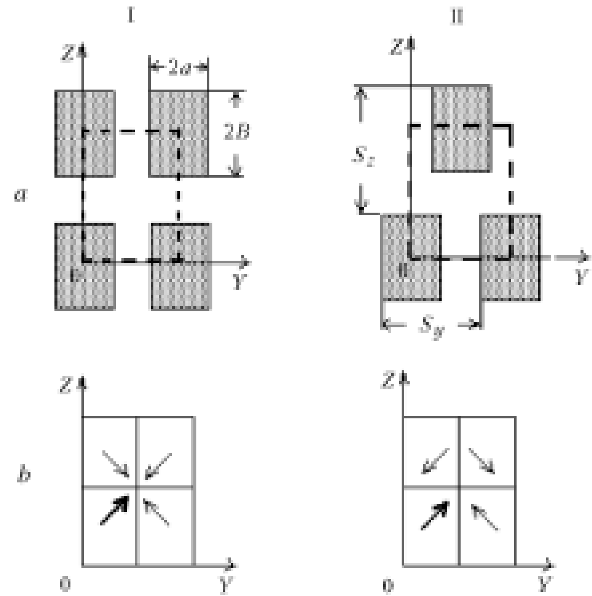


Fig. 1. Diagram of arrangement of cylinders in the rectangular (I) and oblique (II) order (a) and the symmetry of the field distribution in the computational region (b).

$$\varphi = -\,\mathbf{h}^2 - P\,(\mathbf{eh})\,, \quad \mathbf{f}_\mathrm{m} = -\,\mathrm{sign}\,(\Delta\chi)\,\nabla\varphi\,. \tag{2}$$

It is noteworthy that the dimensionless magnetophoretic potential φ in (2) has been determined for paramagnetic particles ($\Delta\chi > 0$), i.e., paramagnetic particles move in the direction of the φ minimum, whereas diamagnetic ones move in the direction of the maximum.

We determine the eigenfield of the structure by summation of the contributions from a fairly large number of individual cylinders. Due to the periodicity of the structure, the field distribution in it is also periodic and is completely characterized by the distribution in the regions bounded by dashed lines in Fig. 1a. Moreover, in the computations, it is sufficient to consider a quarter of the above regions (Fig. 1b). Performing computations in the quarter marked by a heavy arrow, we can obtain the vectors of the field in the remaining quarters by symmetry transformations by which the heavy arrow is transformed to thin arrows in Fig. 1b.

We consider a finite set of cylinders which consists of $2M + 1$ layers; each layer contains $2M + 1$ cylinders. The origin of coordinates is located at the center of the central cylinder. The positions of the cylinders' axes are prescribed using a pair of integers j and k running through the values from $-M$ to M:

$$Y_{jk} = S_y \left(j + \frac{1}{2}\,\alpha \cdot \mathrm{odd}\,(k) \right), \quad Z_{jk} = kS_z\,. \tag{3}$$

Here we have $\alpha = 0$ for the rectangular arrangement of the cylinders and $\alpha = 1$ for the oblique arrangement; the function odd (k) takes on a value of 0 in the case of even k and a value of 1 in the case of odd k.

We find the magnetic field and the magnetophoretic potential in the region $0 \le Y \le S_y/2$ and $0 \le Z \le S_z/2$. After the computation of the field $\mathbf{h}_0(Y, Z)$ of the central cylinders, we can find the field of any other cylinder according to

$$\mathbf{h}_{jk}\,(Y, Z) = \mathbf{h}_0\,(Y - Y_{jk}, Z - Z_{jk})\,. \tag{4}$$

Using the cylinder half-width a as the distance scale, we determine the strength of the field of the central cylinder at an arbitrary point prescribed by the dimensionless radius vector $\mathbf{r}(x, y, z)$, where $x = X/a$, $y = Y/a$, and $z = Z/a$, by the relation

$$\mathbf{h}_0(\mathbf{r}) = -\frac{1}{2\pi}\int_{-b}^{b}\int_{-1}^{1}\int_{-\infty}^{\infty}\frac{1}{|\mathbf{r}-\mathbf{r}_0|^3}\left[\mathbf{e} - 3\,\frac{[\mathbf{e}\cdot(\mathbf{r}-\mathbf{r}_0)\,(\mathbf{r}-\mathbf{r}_0)]}{|\mathbf{r}-\mathbf{r}_0|^2}\right]dx_0dy_0dz_0\,. \tag{5}$$

Here $\mathbf{r}_0(x_0,\ y_0,\ z_0)$ is the radius vector of the cylinder's points. The computation result will be represented in the form

$$h_{0x}^{\parallel} = h_{0x}^{\perp} = 0\,,\quad h_{0y}^{\parallel} = -\,h_{0z}^{\perp} = N_0\,(y, z;\, b)\,,\quad h_{0y}^{\perp} = h_{0z}^{\parallel} = T_0\,(y, z;\, b) \tag{6}$$

(the results for the case of parallel magnetization of the structure (the external field is guided along the Y axis) are marked by the superscripts $\parallel$ and the results for the case of perpendicular magnetization (the field is along the Z axis) are marked by the superscript $\perp$)

$$N_0\,(y, z;\, b) = \frac{1}{\pi}\left[\arctan\frac{1-y}{b-z} + \arctan\frac{1+y}{b-z} + \arctan\frac{1-y}{b+z} + \arctan\frac{1-y}{b+z}\right],$$

$$T_0\,(y, z;\, b) = \frac{1}{2\pi}\ln\frac{[(b-z)^2 + (1+y)^2]\,[(b+z)^2 + (1-y)^2]}{[(b-z)^2 + (1-y)^2]\,[(b+z)^2 + (1+y)^2]}\,.$$

The magnetic field of the ordered set of cylinders is expressed by the relations

$$h_y^{\parallel} = -\,h_z^{\perp} = N\,(y, z;\, b, s_y, l, \alpha, M)\,,\quad h_{0y}^{\perp} = h_{0z}^{\parallel} = T\,(y, z;\, b, s_y, l, \alpha, M)\,, \tag{7}$$

where

$$N = \sum_{k=-M}^{M}\ \sum_{j=-M}^{M} N_0\,(y - y_{jk},\, z - z_{jk};\, b)\,;\quad T = \sum_{k=-M}^{M}\ \sum_{j=-M}^{M} T_0\,(y - y_{jk},\, z - z_{jk};\, b)\,,$$

here, we have

$$y_{jk} = s_y\left(j + \frac{1}{2}\,\alpha\cdot\mathrm{odd}\,(k)\right),\quad z_{jk} = kls_y\,,\quad s_y = \frac{S_y}{a}\,,\quad l = \frac{S_z}{S_y}\,.$$

With account for (7), expression (2) for the magnetophoretic potential of the system in question takes the form

$$\begin{aligned}\varphi^{\parallel}\,(y, z;\, b, s_y, l, \alpha, M, P) &= -\,T^2 - N^2 - PN\,,\\ \varphi^{\perp}\,(y, z;\, b, s_y, l, \alpha, M, P) &= -\,T^2 - N^2 + PN\,.\end{aligned} \tag{8}$$

Square Cylinders. The distribution of the magnetophoretic potential in the system in question is determined by a large number of parameters: the shape of the cylinders (parameter b), the relative steps of the ferromagnetic structure in the horizontal (s_y) and vertical (s_z) directions, the way of arrangement of the layers (parameter α), the direction of magnetization, and the dimensionless field strength P. We consider a system of square cylinders (b = 1) in detail. Figure 2 shows the potential distribution of this system for the values l = 1 and P = 4. Constant-level lines for compacted (s_y = 3), normal (s_y = 4), and rare (s_y = 5) structures are presented; the distances between the cylinders in a layer and between layers are one, two, and three half-widths of a cylinder. The color intensity of the isolines builds up with decrease in the corresponding algebraic value of the potential. Paramagnetic particles move in the direction of buildup in the color intensity of the isolines, whereas diamagnetic particles move in the opposite direction. The potential difference between neighboring isolines in this figure (and in all subsequent figures) has nearly the same value: $\Delta\varphi \approx 0.25$. Rectangular layerings (α = 0) are presented only for the cases of longitudinal magnetization. By virtue of symmetry, transverse magnetization is obtained by simple rotation of the picture by an angle $\pi/2$.

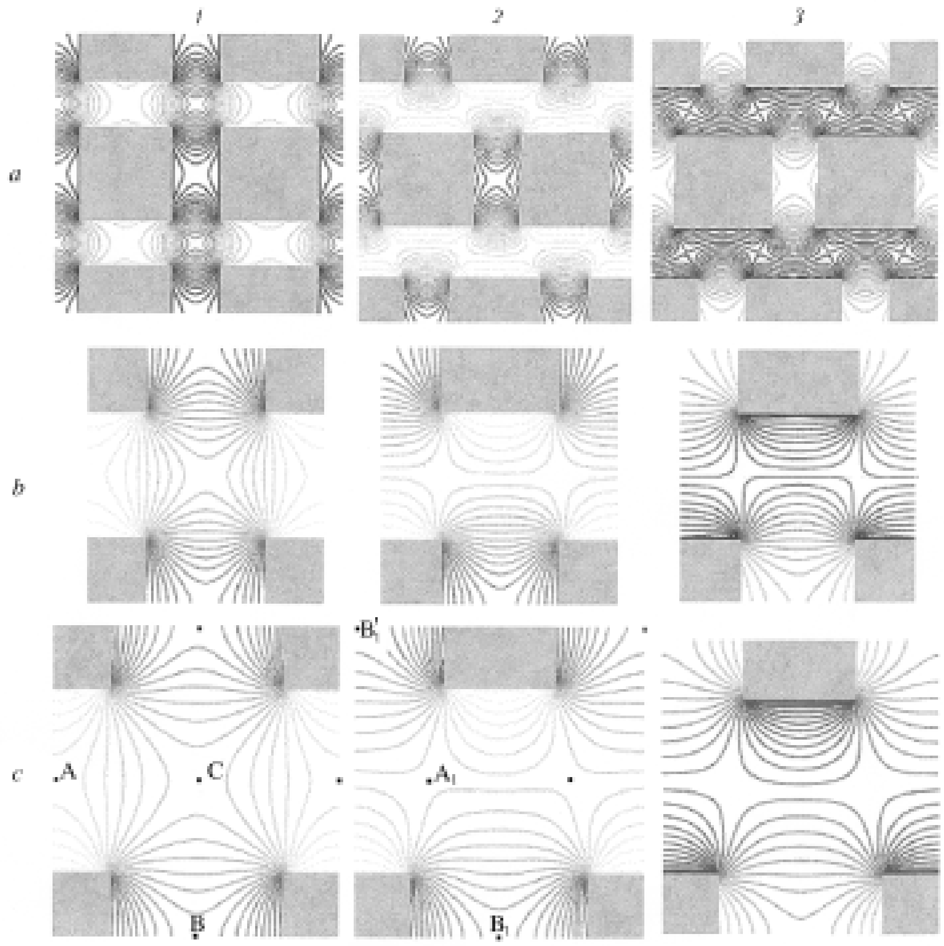


Fig. 2. Isolines of magnetophoretic potential of the system of square ferrocylinders in the case of their different arrangements and different directions of magnetization (P = 4 and b = 1): 1) rectangular systems magnetized in parallel; 2 and 3) oblique systems magnetized in parallel and perpendicularly; a) compacted arrangement (s_y = 3), b) normal arrangement (s_y = 4), and c) rare arrangement (s_y = 5).

Noteworthy are certain general features of the magnetophoretic field. Paramagnetic particles move from the suspension volume to the normal cylinder surfaces, whereas diamagnetic particles move to those tangential to the external field. The magnetophoretic force is maximum at the cylinders' angles where the potential isolines are bunched. Comparing the rectangular (Fig. 2 (1)) and oblique (Fig. 2 (2, 3)) arrangements, we note that the latter ensures a more uniform distribution of the isolines (and consequently the magnetophoretic force) in the space between cylinder layers.

Under the action of the magnetophoretic force, particles follow the trajectories that are perpendicular to the potential lines; a substantial influence on the process of isolation of particles may be exerted by the presence of unstable-equilibrium points at which the gradient of magnetophoretic potential vanishes. The location of such points is clear from symmetry considerations (see Fig. 1c and Fig. 2c). The motion of a particle in the vicinity of the equilibrium point slows down; consequently, the isolation of particles from the part of the volume traversed by a bundle of trajectories approaching equilibrium points slows down, too.

We consider the trajectories of movement of particles from the cylinder surface $z = b$ to the settling surface and determine the movement time as a function of the initial position of a particle on the cylinder surface.

The motion of the particle is prescribed by the equation of balance of viscous and magnetophoretic forces. Writing the viscous force in the Stokes approximation $\mathbf{f}_\eta = -3\pi\eta d(d\mathbf{R}/dt)$ and using the scale of the distance a and the time

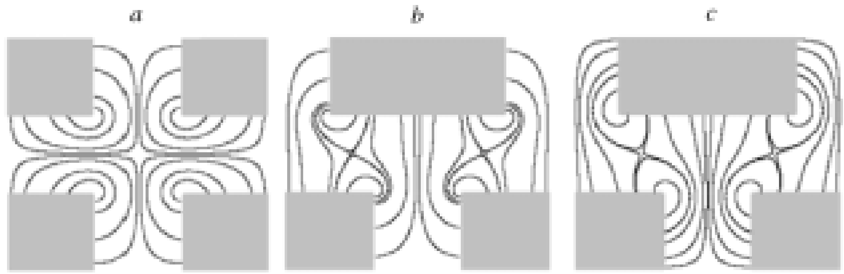
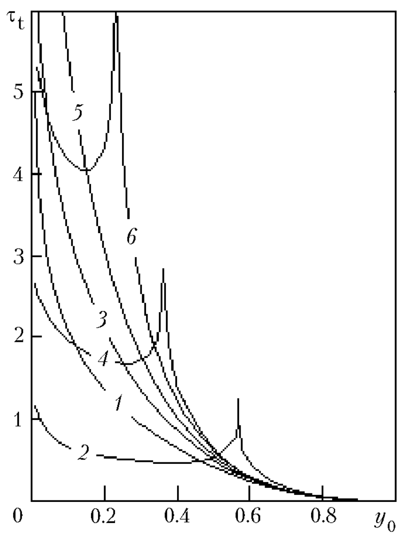


Fig. 3. Trajectories of particles for the parametric values adopted in constructing the potential isolines in Fig. 2a: 1, 2, and 3) respectively a, b, and c.



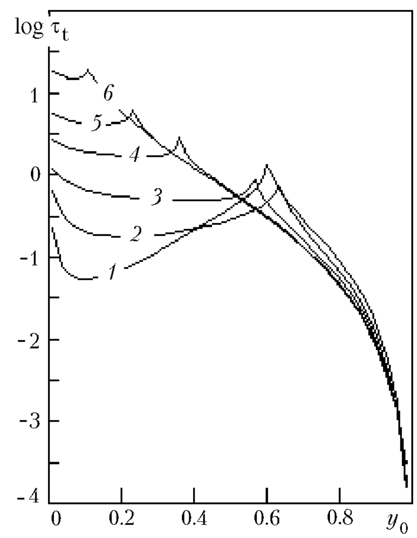


Fig. 4. Time τ_t of movement of a paramagnetic particle to the settling surface vs. initial position y_0 on the surface $z = b$ for the parametric values adopted in constructing the potential isolines in Fig. 2a: 1, 3, and 5) rectangular arrangement of the cylinders; 2, 4, and 6) oblique arrangement. s_y = 3, 4, and 5.

Fig. 5. Time τ_t of movement of a paramagnetic particle to the settling surface vs. initial position y_0 on the surface $z = b$ for the case of the oblique arrangement of the cylinders for b = 1, l = 1, P = 4, and different densities of the structure: 1) s_y = 2.25; 2) 2.5; 3) 3; 4) 4; 5) 5; 6) 7.

$$t^* = \frac{3\pi d a^2}{|\Phi^*|} = \frac{9a^2}{|\Delta\chi|\, d^2 (\pi I_s)^2}\,,$$

we obtain the equation of motion in dimensionless form:

$$\frac{d\mathbf{r}}{d\tau} = -\,\mathrm{sign}\,(\Delta\chi)\,\nabla\varphi\,. \tag{9}$$

The problem formulated is reduced to solution of Eq. (9) when $\Delta\chi > 0$ for longitudinal magnetization and $\Delta\chi < 0$ for transverse magnetization with the initial condition $y(0) = y_0$, $z(0) = b$. We find the solution by the finite-difference method on the time grid $\tau_i = i\Delta\tau$. Introducing the notation $y_i = y(\tau_i)$ and $z_i = z(\tau_i)$, we have

$$y_{i+1} = y_i - \mathrm{sign}\,(\Delta\chi)\,\varphi_z\,(y_i, z_i)\,\Delta\tau\,,\quad z_{i+1} = z_i - \mathrm{sign}\,(\Delta\chi)\,\varphi_z\,(y_i, z_i)\,\Delta\tau\,.$$

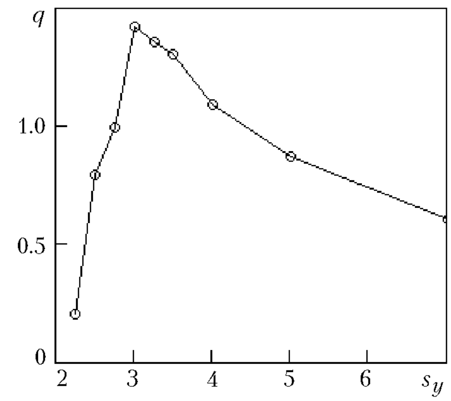


Fig. 6. Parameter of efficiency q of the filter system vs. density of laying of the ferrocylinders s_y for $b = 1$, $l = 1$, and $P = 4$.

Figure 3 shows the trajectories of a particle for the values of the parameters which have been adopted in constructing the potential isolines in Fig. 2a (1, 2, and 3). It is seen that in the case of rectangular arrangement of the cylinders particles move from one surface of the same cylinder to another. The exchange of particles between the cylinders in the neighboring layers is observed in the case of the oblique arrangement. The time τ_t of movement of a paramagnetic particle to the settling surface as a function of the initial position y_0 on the surface $z = b$ in parallel magnetization for the values of the parameters which have been adopted in constructing the potential isolines in Fig. 2a (1 and 2), 2b (1 and 2), and 2c (1 and 2) is plotted in Fig. 4. The time τ_t is determined at the instant of satisfaction of the conditions $y < 1$, $z < b$ or $y > s_y/2 - 1$, $z > s_y l - b$. As follows from the dependences presented, the magnetophoretic isolation of particles in the oblique and rectangular systems is significantly different in character. A distinctive feature of the rectangular system is a sharp increase in the isolation time with decrease in y_0. For small y_0, a particle in the rectangular system travels in the vicinity of three equilibrium points (points A, B, and C in Fig. 2c (1)). Slowed-down motion of the particle in the vicinity of the equilibrium points must give rise to stagnation zones the isolation of particles from which is difficult and may substantially increase the time and decrease the depth of cleaning. When the arrangement of the cylinders is oblique, a particle can meet with only one stagnation zone (in the vicinity of either point A_1 or point B_1' in Fig. 2c (2)). The occurrence of the peak on the $\tau_t(y_0)$ curve for a certain value y_0^* is associated with the latter point. The trajectories beginning when $y_0 < y_0^*$ terminate on the cylinder from the neighboring row, whereas those beginning when $y_0 > y_0^*$ terminate on their own cylinder. Due to the decrease in the number of stagnation zones, on the one hand, and the decrease in the trajectory length for $y_0 < y_0^*$, on the other, the oblique system must have much better separation parameters.

We consider (Fig. 5) the dependence $\tau_t(y_0)$ for the oblique system in a wider range of values of the structural steps s_y for constant values of the remaining parameters ($b = 1$, $l = 1$, and $P = 4$). A fundamental conclusion drawn from the results presented is that with decrease in the step below $s_y = 3$ the influence of the equilibrium point begins to increase, which is manifested as the increase in the height and width of the peak on the $\tau_t(y_0)$ curve. This fact points to the existence of the optimum value of a structural step. As the criterion of efficiency of the magnetic structure of the filter we introduce the parameter q, i.e., the ratio of the cross-sectional area of the interpolar space per cylinder to the characteristic separation time, for which we take the peak value τ_t^* on the dependence $\tau_t(y_0)$:

$$q = \frac{1}{4\tau_t^*}\left(s_y^2 l - 4b\right). \tag{10}$$

The dependence $q(s_y)$ calculated for $b = 1$, $l = 1$, and $P = 4$ (Fig. 6) shows that the optimum structural step is $s_y = 3$.

Elongated Cylinders. In actual practice, a magnetic field of strength about 20 kOe is necessary for saturation magnetization of a square cylinder made of soft-magnetic steel. The establishment of such a field involves certain difficulties building up with the dimensions of the system. In this connection, elongated cylinders magnetizable along their long side are of interest. We consider systems of such cylinders in which the height is either twice as large as the width ($b = 2$) or half as large ($b = 0.5$). Such a structure is close to magnetic saturation even for the dimension-

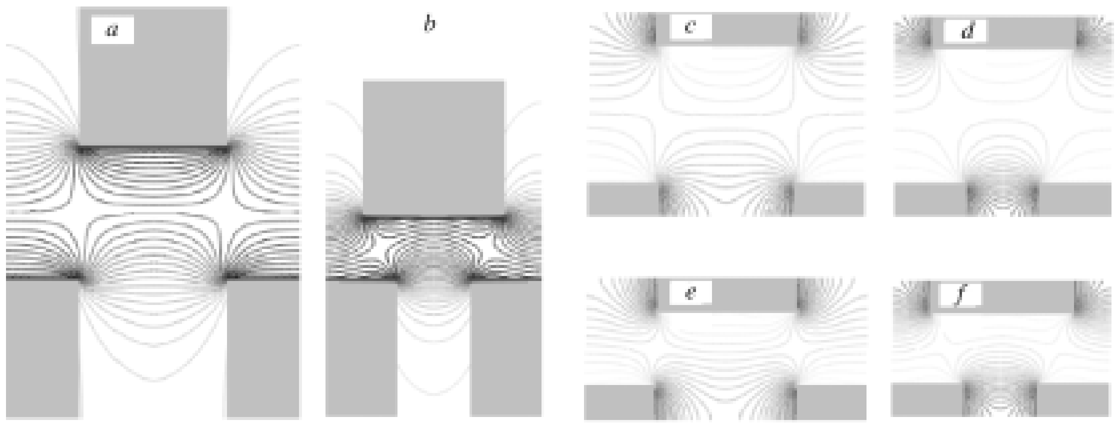


Fig. 7. Magnetophoretic-potential distribution in oblique systems of elongated ferrocylinders magnetized along their long side at $P = 4$ and different values of the geometric parameters: a) $b = 2$, $s_y = 4$, and $s_z = 6$; b) 2, 3, and 4; c) 0.5, 4, and 3; d) 0.5, 3, and 3; e) 0.5, 4, and 3; f) 0.5, 3, and 4.

less field strength $P = 2$. Examples of the magnetophoretic-potential distribution are presented in Fig. 7 for the normal and compacted layings for $P = 4$.

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NOTATION

a, half-width of a cylinder, cm; *B*, half-height of a cylinder, cm; *b*, cylinder height-to-width ratio; *d*, particle diameter, cm; **e**, unit vector in the direction of the external magnetic field; $\mathbf{F}_m$, magnetophoretic force, cm·g·sec^{-2}; $\mathbf{f}_m$, dimensionless magnetophoretic force; $\mathbf{H}_0$ and $\mathbf{H}'$, strength of the external magnetic field and of the eigenfield of ferrocylinders, Oe; **h**, dimensionless strength of the magnetic eigenfield of ferrocylinders; I_s, saturation magnetization, G; $l = S_z/S_y$; *P*, dimensionless strength of the external field; *q*, parameter (determined by relation (10)) of efficiency of the magnetophoretic structure; **r**, dimensionless radius vector; *R*, radius vector, cm; S_y and S_z, steps of laying of ferrocylinders along the directions *y* and *z*, cm; s_y and s_z, dimensionless steps of laying of ferrocylinders (in units of *a*); *t*, time, sec; *v*, particle volume, cm^3; *X*, *Y*, *Z*, Cartesian coordinates; *x*, *y*, *z*, dimensionless Cartesian coordinates; *N* and *T*, functions of a point; α, parameter characterizing the mutual position of ferrocylinder layers; η, viscosity, P; τ, dimensionless time; φ, dimensionless magnetophoretic potential; χ, magnetic susceptibility. Subscripts and superscripts: 0, initial, t, movement (travel); $\perp$, perpendicular; $\|$, parallel; m, magnetophoretic; s, saturation.

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